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# Joint Time-Frequency Analysis of a 19 Story Instrumented Building During Two Earthquakes

C.J. Black[1], C.E. Ventura[2]

## ABSTRACT

Strong motion records obtained from a 19 story, steel frame building in the Los Angeles area are analyzed using both frequency domain and joint time-frequency domain techniques. The strong motion records were recorded during the 1971 San Fernando and the 1994 Northridge earthquakes. This paper illustrates how joint time-frequency analysis can improve the understanding of the seismic response of structures. A joint-time frequency analysis allows the peaks in the frequency response function to be tracked over time and thus, the identification of the temporal location of response or the presence of a frequency shift. This paper presents a brief introduction to analysis in the joint time-frequency domain as well as an example of its application.

## INTRODUCTION

A frequency domain analysis of strong motion records obtained at the 1901 Avenue of the Stars Building, a 19 story, steel frame building in the Los Angeles area, was recently conducted by Black and Ventura (1998). This study showed that the fundamental, transverse mode of the building remained relatively constant throughout the two earthquakes studied, while the fundamental longitudinal mode shifted slightly. A detailed joint time-frequency analysis was conducted as part of a Masters Thesis by Black (1998), in order to study the observed shift. The information presented in this paper highlights the results of these previous studies.

The two earthquakes used in the analysis were the San Fernando and the Northridge earthquakes. The San Fernando earthquake occurred on February 9, 1971 at 7:00 am PST. The hypocenter was about 27 km from downtown Los Angeles with a focal depth of 13 km. The earthquake lasted approximately 45 seconds and had a magnitude of $M_L = 6.4$. The Northridge earthquake occurred on January 17, 1994 at 4:31 am PST. The hypocenter was about 32 km West-Northwest of Los Angeles with a focal depth of 19 km. The Northridge earthquake lasted approximately 45 seconds and had a magnitude of $M_L = 6.4$.

The epicenter of the Northridge earthquake was located 20 km from the 1901 Avenue of the Stars building and the peak ground acceleration at the site was 0.32g. The epicenter of the San Fernando earthquake was approximately 39 km, with a peak ground acceleration of 0.15 g's.

This paper presents a brief overview of the different methods available for joint time-frequency analysis with emphasis on the Short Time Fourier Transform (STFT) method. This is followed by a description of the building and the results of the joint time-frequency analysis performed.

## JOINT TIME-FREQUENCY ANALYSIS

Joint time-frequency analysis involves the analysis of a signal in both the time and frequency domains simultaneously. There are two basic approaches to analysis in the joint time-frequency domain. The first approach is to initially cut the signal into slices in time and examine their frequency content. The second approach involves the filtering of discrete frequency bands which are in turn, sliced into discrete time bands and analyzed for their energy content. The first approach describes the STFT and Cohen's Class functions, while the latter describes the Wavelet Transform method.

Many of the Cohen's Class functions, including the Wigner distribution however, do not appear to be easily applied to the analysis of civil engineering data. They are more accurate than other methods in terms of energy, but generally they are not manifestly positive. Negative values in the time-frequency plane complicate the analysis and limit the physical significance of the results. The Short Time Fourier Transform, which can also be expressed by the Cohen Class

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1. Graduate Student, Dept. of Civil and Environmental Engineering, The University of California, Berkeley. Research conducted while a Graduate Student in the Dept. of Civil Eng., The University of British Columbia.
2. Assoc. Professor, Dept. of Civil Engineering, The University of British Columbia 2324 Main Mall, Vancouver, B.C. V6T 1Z4

function, is a manifestly positive distribution. As well as this, it has the advantage of being based on a fundamental analysis tool, the Fourier Transform. Detailed information on the application of the different methods for joint time-frequency analysis of strong motion data is given by Black (1998).

The analysis presented in this paper makes use of the STFT to obtain the time-frequency spectrogram. The spectrogram gives the amplitude of a signal in the time and frequency domains simultaneously. A similar study with emphasis on the joint time-frequency analysis procedure was conducted on a 20 story reinforced concrete structure in the Los Angeles Area (Black and Ventura, 1999).

## The Time-Frequency Response Function

If the input into a structure is known, the Time Frequency Response Function (TFRF) can be used to estimate the natural frequencies of the structure. It is similar to the Frequency Response Function (FRF) which is the response of the structure to a unit amplitude harmonic function. The TFRF however, provides information in both the time and frequency domains simultaneously.

A time-frequency spectrogram is obtained via the STFT. That is, the original signal, in this case the measured accelerations, are analyzed using the Fourier Transform with a moving window. In the analysis presented in this paper the window function chosen was a 256-point Hanning window.

Consider a viscously damped single-degree-of-freedom (SDOF) system subjected to a known excitation. If $P(\omega, t)_{in}$ represents the time-frequency spectrogram of the input to this system and $P(\omega, t)_{out}$ represents the system's output time-frequency spectrogram, then the TFRF is defined as:

$$TFRF(\omega, t) = \frac{P(\omega, t)_{out}}{P(\omega, t)_{in}} \tag{1}$$

It has been shown by Black (1998), that when the Hanning window is used to define the time intervals where the data is to be analyzed, the TFRF approximates the square of the FRF. The frequency content at a particular time of the selected time interval, $t$ is given by:

$$TFRF_t(\omega) \approx \left\{\frac{1}{(k - \omega^2 m + i\omega c)}\right\}^2 . \tag{2}$$

As is commonly done for the system identification of a multi-degree-of-freedom system by frequency domain analysis of strong motion records, the TFRF defined above can also be used to determine instantaneous values of the natural frequencies of the system throughout the duration of the shaking.

It should be noted that because of the presence of "noise" in recorded data, the results obtained from the analysis of strong motion data are generally not as "clean" as is the case of an analytical SDOF system. The following points should be considered when analyzing time-frequency results resulting from the application of Equation 2.

The TFRF of strong motion records may consist of a series of peaks rather than a stable ridge. This is because the structure vibrates in more than one mode and at a particular time a particular mode may not be excited or may have a negligible contribution to the response at that particular time. The amplitude of the TFRF does not indicate the presence of heavy shaking. It rather indicates which frequency components are present at a certain time and their relative strengths.

The information contained in the TFRF is fundamentally different than the FRF. The FRF averages the frequency content over the entire time of the record. More specifically, it is the ratio of the total energy contained in the output, at a particular frequency, over that in the input. The TFRF conveys information on the relative energy levels, at a particular frequency, during a small window of time. For this reason, the magnitude of the TFRF may be misleading. When the magnitude of shaking at the roof level, for a given time and frequency, is greater than that at the base, a peak will be shown. Hence, a large peak may be present during free decay of the vibration when the input is very small.

## DESCRIPTION OF THE 1901 AVENUE OF THE STARS BUILDING

The Avenue of the Stars building is located in the Century City area of Los Angeles at 1901 Avenue of the Stars. This moment resisting, steel frame, office building has 19 stories above the ground level with four parking levels below ground. The floor plan of the 1901 building (above ground) measures 240 feet wide by 110 feet deep (Murphy, 1973). A Photo of the 1901 building is shown in Figure 1.

The soil conditions at the site are generally fine sand. The building's foundation is driven steel I-beam piles under the main structural tower and spread footings elsewhere. The lateral load resisting system consists of four ductile steel moment-resisting frames in the major axis direction and 5 X-braced steel frames in the minor axis direction (Murphy, 1973).

Damage during the San Fernando earthquake was not severe. During the Northridge earthquake bracing elements at the penthouse buckled and signs of distress and motion along the brace lines in upper floors were observed. Minor non-structural and content damage was observed but operation resumed within one day (John A. Martin and Associates, 1997).

## RESULTS OF JOINT TIME-FREQUENCY ANALYSIS

The time-frequency information presented in this paper is displayed in a joint time-frequency plot which contains the input and output time signals, the standard FRF and the TFRF. The TFRF is shown as a contour plot. The contours have been limited to 3 levels for simplicity. A legend of the contour levels is shown in the top left corner of each plot. The TFRF is cut at the highest contour level, which does not necessarily correspond to the peak magnitude of the TFRF. The width of the contour however, conveys information about the peak. A contour with small width means the magnitude is close to the peak. The spacing of the contours indicates the sharpness of the peak.

At a sampling rate of 100 sps, the length of a windowed signal, with a window size of 256 points, is 2.56 seconds. The fundamental period of the building is 3.4 seconds and thus the windowed signal does not completely capture one full oscillation of the structure's fundamental period. For this reason, there may be slight distortion in the frequency content estimation; however no direct evidence of this was observed in this study (see Black, 1998).

Transverse Direction

As mentioned earlier, a standard frequency domain analysis was conducted by Black and Ventura (1998) in which it was shown that the fundamental frequencies do not shift dramatically between the two earthquakes. Table 1 presents the fundamental frequencies in the two directions. The TFRFs for the transverse direction show stable frequencies for both earthquakes (Black, 1998). This is expected as the frequency domain analysis showed little variation in the calculated frequencies.

Longitudinal Direction

The frequency domain analysis of records in the longitudinal direction showed that there is a shift in the fundamental longitudinal frequency between the two earthquakes. The TFRF calculated from the Northridge records, given as Figure 2, shows that the fundamental longitudinal mode has a relatively constant peak at 0.22 Hz. The TFRF for the San Fernando records however, does show a shift.

Figure 3 shows a gradual decrease from 0.29 Hz to 0.22 Hz. The FRF appears to be dominated by the strong peak during the first 5 seconds of shaking. Figure 4 presents a close up of the time-frequency plot for the San Fernando earthquake in the frequency range 0 to 0.6 Hz. The TFRF appears to have two dominant frequencies, one around 0.32 Hz and one at 0.25 Hz. The FRF however, only indicates one peak at 0.29 Hz. The FRF averages the peaks over time and thus it is possible that the two dominant frequencies combined in such a way that only a single peak is visible in the frequency domain. The fundamental mode in the transverse direction is 0.32 Hz. If coupling exists between the transverse and longitudinal directions, the fundamental mode in the transverse direction could be influencing the FRF in the longitudinal direction. The combination of two frequencies which yield a single peak is sensitive to the duration of the two signals, the sampling rate and the amplitude of the signals. With the right combination of the above factors, it can be shown that a time signal with two distinct frequency components can average to form a single peak in the frequency domain.

One interesting feature of the TFRF is its ability to provide insight into the behavior of the FRF. For example, the frequency fluctuations observed in the TFRFs around the second natural frequency may explain the spreading in the peak of the FRF. As well it is seen that the first mode does not dominate the response during heavy shaking.

For the case of a steady-state forced input, a structure will vibrate at the same frequency as the driving force. An earthquake however is transient and steady state motion would not be achieved. For this reason, there will be some modal response of the building during heavy shaking. The low level of modal response seen in the TFRF does not necessarily mean that the building does not vibrate in its modes during heavy shaking, rather that the magnitude of the response, at a particular frequency, is eclipsed by the magnitude of the input.

## CONCLUSION

The frequency domain analysis of the strong motion records of the building identified a shift in the fundamental longitudinal mode. The reason for this shift is not decipherable from frequency domain analysis alone. A joint time-frequency analysis shows that the shift is not due to heavy shaking but that the peak at 0.29 Hz is actually a combination of two modes; the fundamental longitudinal mode vibrating at 0.22 Hz and the fundamental transverse mode at 0.32 Hz. This also indicates that these modes have significant three dimensional components.

The Time-Frequency Response Function, presented in this paper allows for the study of many aspects of the dynamic behavior of structures not easily studied with frequency domain analysis alone. This includes the coupling between modes, the temporal location of modal response and the identification of frequency shifting, or lack of it.

## ACKNOWLEDGMENTS

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## REFERENCES

Black, C.J. (1998), *Dynamic Analysis of Civil Engineering Structures Using Joint Time-Frequency Methods*. M.A.Sc. Thesis, The University of British Columbia.

Black, C.J. and Ventura, C.E. (1998), Seismic Response of Four Instrumented Buildings During two Earthquakes. *Proceedings of the XVIth International Modal Analysis Conference*, Santa Barbara, California.

Black, C.J. and Ventura, C.E. (1999), Joint Time-Frequency Analysis of a 20 Story Instrumented Building During Two Earthquakes. *Proceedings of the XVIIth International Modal Analysis Conference*, Kissimee, Florida.

Cohen, L. (1995), *Time-Frequency Analysis*. Prentice Hall PTR, Englewood Cliffs, New Jersey.

Murphy, L.M. (1973), *San Fernando, California, Earthquake of February 9, 1971*. US Department of Commerce. National Oceanic and Atmosphere Administration. Washington, D.C.

John A. Martin and Associates, Inc (1997), *Instrumented Buildings Information System*, January 17, 1994, Northridge, California Earthquake. JAMA Earthquake Engineering Series.

| Mode | San Fernando | Northridge |
|---|---|---|
| Fundamental Transverse | 0.29 Hz | 0.32 Hz |
| Fundamental longitudinal | 0.29 Hz | 0.22 Hz |

**Table 1: Fundamental Frequencies Obtained from Strong Motion Records.**

(Adapted from John A. Martin and Associates, 1997)

**Figure 1: Photo of the 1901 Avenue of the Stars Building**

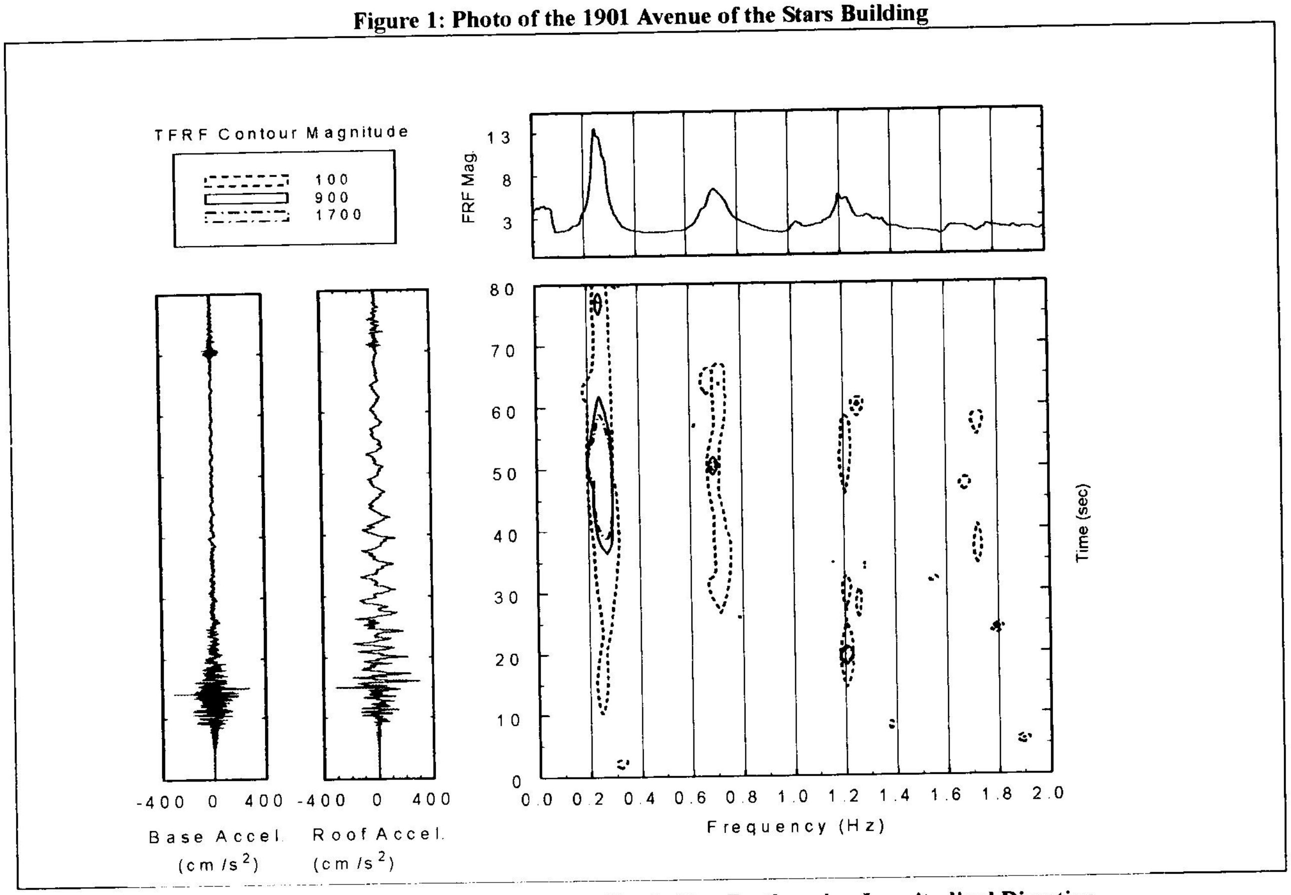


**Figure 2: Joint Time-Frequency Plot - Northridge Earthquake- Longitudinal Direction**

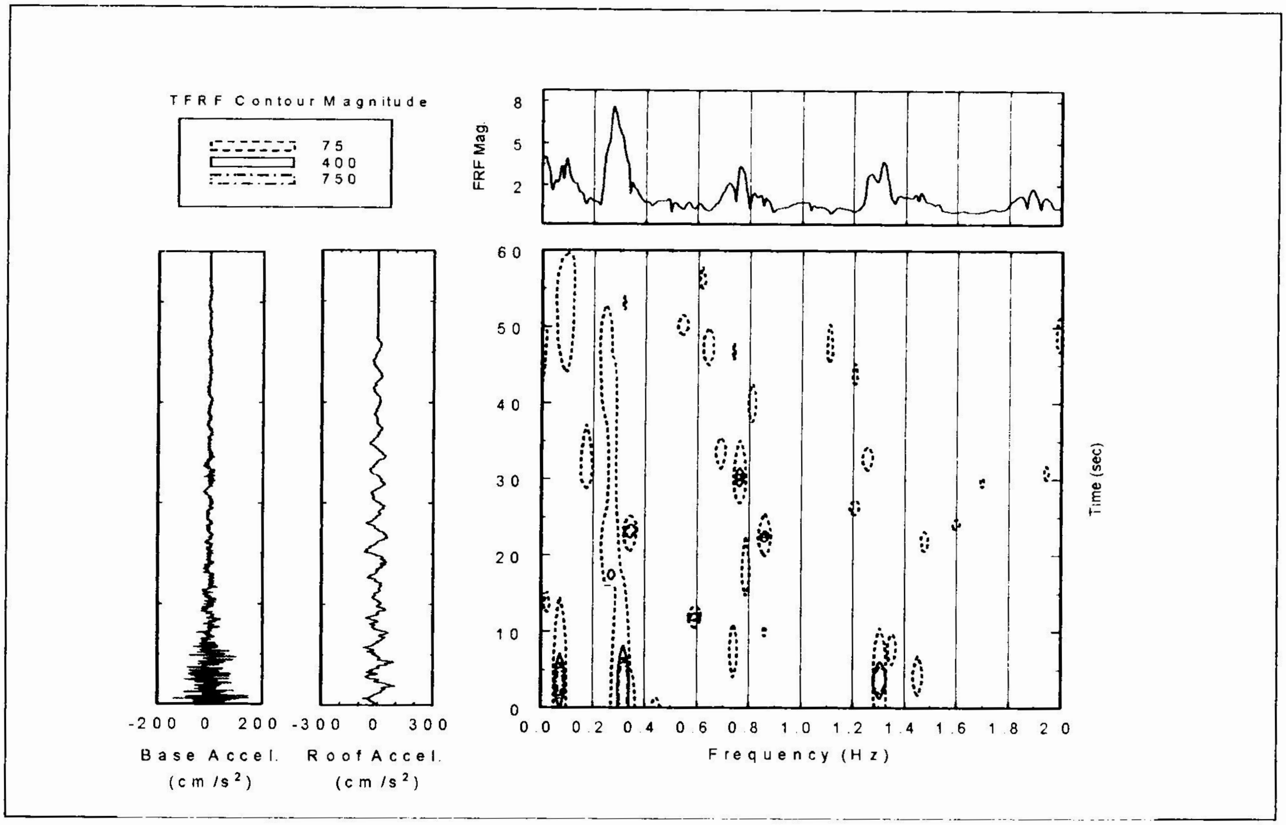


**Figure 3: Joint Time-Frequency Plot - San Fernando Earthquake - Longitudinal Direction**

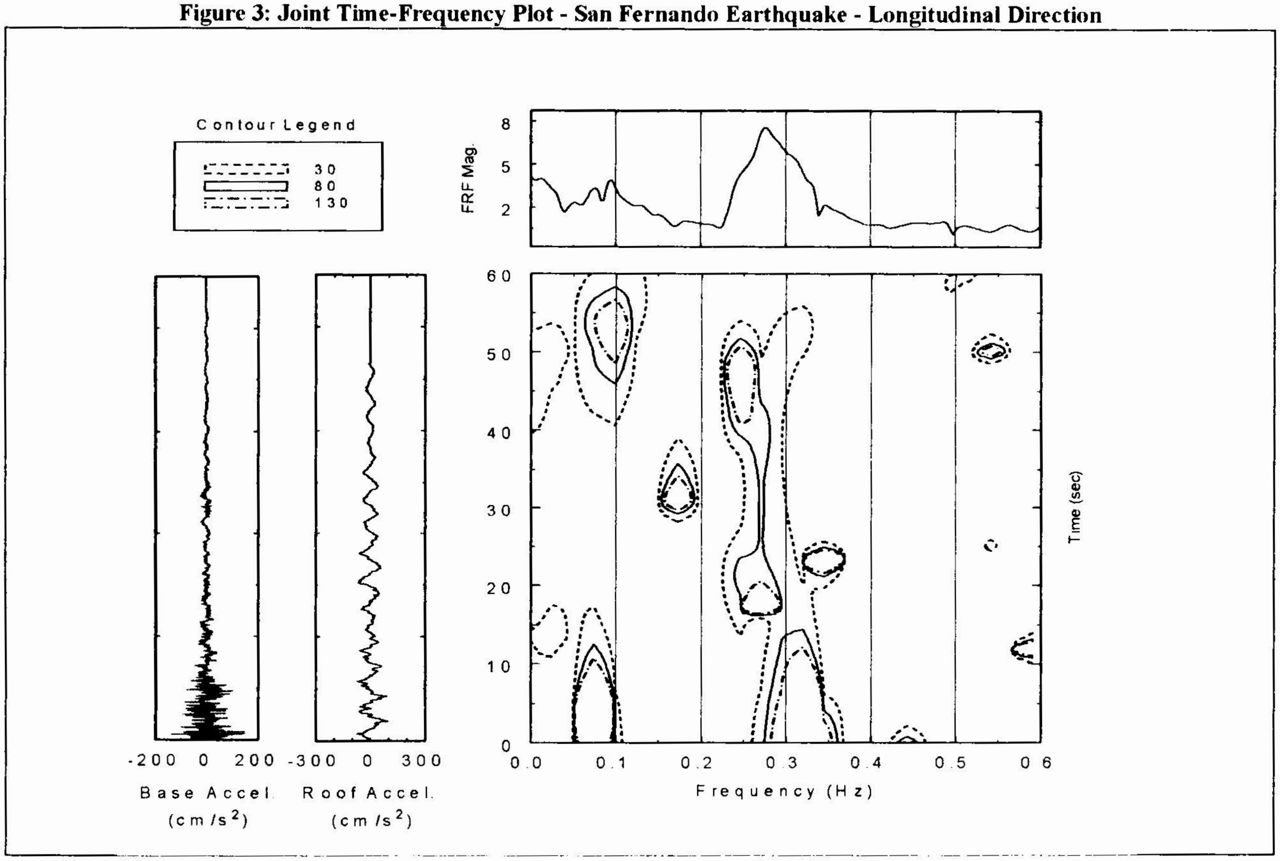


**Figure 4: Joint Time-Frequency Plot - San Fernando Earthquake- Longitudinal (0 to 0.6 Hz)**